Top-Bottom Color and Weak Doublets Seesaw[*]

Marko B. Popovic[†]
Department of Physics, Boston University,
590 Commonwealth Ave., Boston MA 02215

February 1, 2008

Abstract

A brief overview of some of the last decade's developments in the Top-mode/Topcolor class of models is presented. In addition, a new Topcolor type scenario, named Top-Bottom Color, is suggested. A seesaw mechanism with new weak doublet quarks has been introduced to lower the previous theoretical prediction for the large top quark mass; without a seesaw mechanism and with a low scale of Nambu - Jona-Lasinio triggering interactions, the top quark mass was predicted to be $O(500\text{GeV})$. In Top-Bottom Color an effective, composite two Higgs doublet model is obtained where the third generation isospin splitting is introduced via tilting interactions related to the broken non-abelian gauge groups (i.e. without strong, triviality-sensitive $U(1)$ groups). To complete discussion a few notes on the fine tuning related to the bottom quark mass are appended in an *addendum*. A generic problem (*Why does the top Yukawa coupling equal one?*) of the Top-mode/Topcolor class of models is pointed out as well. Furthermore, previous work on the third generation seesaw mechanism with weak doublet quarks and dynamical top mass production in the Topcolor spirit is discussed.

---

1   Introduction: Top-mode and Topcolor models in the last decade - a brief (and incomplete) overview

Taking the four-dimensionality of our physical space for granted necessarily yields to the famous hierarchy problem in the standard (non-supersymmetric) gauge theories. The Higgs scalar is quadratically unstable against radiative corrections, and one naturally expects its mass to be of the same (or similar) order as the highest scale of the theory (presumably the Plack scale $\sim 10^{19}$GeV) [1].

Models with strong dynamics, like Technicolor (TC) [2], address the hierarchy problem through the slow, logarithmic running of gauge couplings of non-abelian gauge groups that, after a huge interval of energy scales, finally become strong enough to produce substantial, "interesting", effects on our low-energy world. The main effect is the presence of *condensates* - strongly interacting *new* fermions bind to form condensates that break the global symmetries giving Nambu-Goldston bosons with appropriate quantum numbers (to be *eaten* by Z and W); these theories do not posess a *fundamental* Higgs field floating at the highest energy scales.

Moreover, the fermion masses can be explained in an extended framework of this dynamical scheme - in the spirit of Extended Technicolor (ETC) [3]. The standard model (SM) fermions obtain their masses through Yukawa type couplings with now 'composite scalar fields', implying the presence of four-fermion interactions (from broken ETC, i.e. extended TC gauge group structure) at higher energies. Although in many ways attractive, this idea carries a number of serious challenges and drawbacks - particularly in the simultaneous dynamical breaking of EW symmetry and the creation of mass for third generation quarks (for a review see [4]).

Attempts to introduce a composite scalar Higgs made of third generation SM fermions are expressed in the class of Top-mode models [5, 6]. The Top-mode scheme [6] suggests the existence of 4-fermion interactions at some high-energy scale Λ, i.e.

$$L_{4-fermion} = \frac{g^2}{m_0^2} \left(\overline{\Psi}_{Li}^a t_{Ra} \right) \left(\overline{t}_R^b \Psi_{Lb}^i \right) \ . \tag{1.1}$$

where $\Psi_L = (t, b)_L$ and where the index i (a, b) labels $SU(2)_W$ $(SU(3)_{QCD})$ elements in the fundamental representation. The mass m_0 is of the same (or similar) order as the scale Λ. The coupling g^2 is given *ad hoc* (without dynamical explanation) and it is assumed large. The interaction lagrangian may be rewritten [6] (without changing the equation of motion) as

$$L_{4-fermion} = -g \left(\overline{\Psi}_L t_R \Phi + h.c. \right) - M_0^2 \Phi^\dagger \Phi \tag{1.2}$$

where the auxiliary *static* scalar field Φ is a *would-be* Higgs scalar field at low-energy. The field Φ gets a gauge invariant kinetic term, a positive contribution to the quadratic term and a negative Φ^4 term through the fermion loops in the block-spin renormalization scheme, while sliding to lower energy scales. The predicted
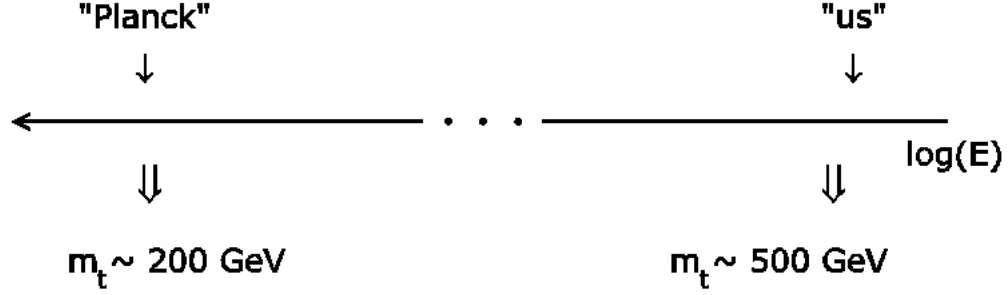
Figure 1: Illustration of the predicted [6] mass of the top quark with respect to the NJL triggering scale.

top quark mass, in the "full SM" analysis [6] with a fixed value of the EW VEV, $v_{EW} \approx 250\text{GeV}$, was found to be $O(500\text{GeV})$ for a triggering interaction at $\Lambda \sim 10^4\text{GeV}$ (with very large, though still benign fine tuning[1]) and $O(200\text{GeV})$ (but still larger than the physical top quark mass) for Λ of the order of the Planck scale[2] (with tremendous fine tuning). We illustrate this result in Fig. 1.

A dynamical basis for the *ad hoc* interaction (1.1) is introduced through the presence of an additional strong $SU(3)$ interaction called Topcolor [9]. The assumed gauge symmetry breaking pattern $SU(3)_A \otimes SU(3)_B \to SU(3)_{QCD}$ produces an octet of heavy colorons [10] that may act as an NJL binding force for the fermions transforming under the stronger "initial" $SU(3)$ gauge group. The recipe is simple - at a scale somewhat below the coloron mass, the coloron exchange diagrams are approximated by four-fermion interactions; fiertzing interactions that couple the lefthanded and righthanded currents in the large N_c limit (N_c is the number of colors, in our case $N_c = 3$) gives the interaction term (1.1).

The Topcolor dynamics was used, combined with TC/ETC dynamics, in third generation specific[3] [11, 12] Topcolor Assisted Technicolor models (TC^2) as well as in the universal[4] [13] TC^2 models in order to create a heavy top quark mass (with top composite scalar VEV, v_t, usually expected to be of order $v_{EW}/3$).

The general motivation for the weak-singlet seesaw models [14, 15] involving third generation quarks and embedded in a Topcolor scheme was to "lower" the

---

[1]Expressed as a small relative deviation of coupling g^2 from the critical coupling g_c^2 (needed to trigger condensation), as predicted for example by the gap equation in the Nambu - Jona-Lasinio (NJL) approximation [7].

[2]This author is not fully convinced of the correctness of the results obtained in the "full SM" analysis [6] because of the specific handling of the composite scalar loops. We agree with reference [6] that scalar loops in principle should be considered (for calculation of, say, the β function) - but only when the effective, scale dependent, mass $m_{Heff}(\Lambda')$ is smaller than the renormalization scale Λ', and not all the way up to the compositeness scale Λ. Therefore, we would rather predict $O(200\text{GeV})$ for the top quark mass when $\Lambda \sim 10\text{TeV}$ (instead of the cited $O(500\text{GeV})$), which follows from the Pagels-Stokar relationship [8] with a fixed value of the top composite scalar VEV,
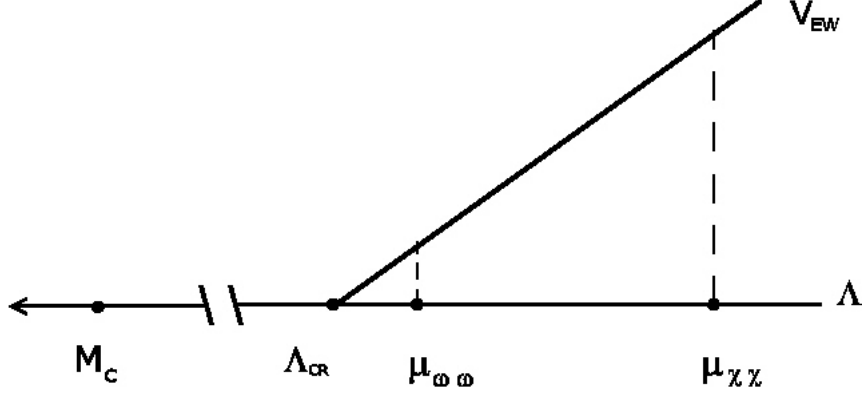
Figure 2: The schematic illustration of the mass renormalization curves in the weak-singlet seesaw Topcolor models [14, 15].

earlier prediction for the top quark mass when $v_t = v_{EW}$ (while keeping the scale of Topcolor low). The seesaw matrix with new weak-singlet quarks χ_L and χ_R was introduced in the following form

$$(\bar{t}_L \ \ \bar{\chi}_L) \begin{pmatrix} 0 & m_{t\chi} \\ \mu_{t\chi} & \mu_{\chi\chi} \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix}. \tag{1.3}$$

The mass term $m_{t\chi}$ is directly related to the Topcolor dynamics (i.e. χ_R and $\Psi_L = (t, b)_L$ feel the stronger "initial" $SU(3)$ gauge group) while for the purpose of our discussion $\mu_{t\chi}$ and $\mu_{\chi\chi}$ may be thought to be the bare masses (though only $\mu_{t\chi}$ may be a fundamental bare mass, i.e. χ_L and t_R feel the weaker "initial" $SU(3)$ gauge group). Sliding down from the triggering scale ($\Lambda < M_c$, where M_c is the coloron mass) the fermion loop induced renormalization dynamics is stopped at the scale $\sqrt{\mu_{t\chi}^2 + \mu_{\chi\chi}^2} \approx \mu_{\chi\chi}$ (assuming $\mu_{\chi\chi} \gg \mu_{t\chi}$). The fermion loop calculation is then used in obtaining the conditions for proper EW symmetry breaking and the mass spectrum of scalar states [15][5]. In addition, taking $m_{t\chi} \approx 600\text{GeV}$ as a working premise demands [15] $m_{\chi\chi}$ be at least 5TeV as implied by the limits on the T parameter. It was suggested [15] that a similar mechanism may be used for the generation of bottom quark mass as well - one may add the new weak-singlet quarks

---

$v_t = v_{EW} \approx 250\text{GeV}$. We plan to address these issues in more detail elsewhere.

[3]Where only third generation quarks feel the stronger "initial" $SU(3)$ gauge group.

[4]Where all SM quarks feel the stronger "initial" $SU(3)$ gauge group.

[5]This author is not fully convinced that the block-spin renormalization approach with the fermion loop contribution alone is justified. It seems that $\mu_{\chi\chi}$ is large enough so that the Φ^4 term contribution to $m_{Heff}(\Lambda')$ may be neglected and that the scalar loop contributions should then be considered at low enough energy scales (larger than $\mu_{\chi\chi}$) - in the framework with a hard momentum cut-off.
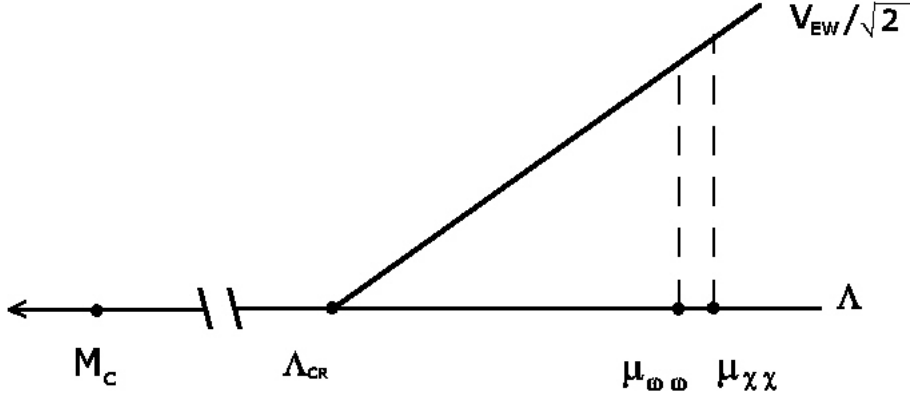
3

Figure 3: The schematic illustration of the mass renormalization curves in the "two-doublet" model [16].

ω_L and ω_R and two new "bare" mass terms $\mu_{\omega\omega}$ and $\mu_{b\omega}$ (with $\sqrt{\mu_{\omega\omega}^2 + \mu_{b\omega}^2} \approx \mu_{\omega\omega}$) to the model structure. With the same NJL triggering interactions in the top and bottom sector, the top and bottom mass renormalization curves share the same path up to the mass $\mu_{\omega\omega}$ where dynamics in the bottom sector stops. This behavior is illustrated in Fig. 2. The scale Λ_{cr} represents the critical scale at which (in the scheme with a hard momentum cut-off, as in [6, 14, 15]), the dynamical fermion mass starts to grow rapidly. The window between Λ_{cr} and $\mu_{\chi\chi}$, in this approximation, is extremely narrow ($O(200\text{GeV})$ for $\mu_{\chi\chi} \approx 5\text{TeV}$). For our purposes the renormalization mass curve may be thought to be linear[6].

Another attempt, reflected in a two-doublet model [16], to introduce the bottom quark mass generation in the Topcolor scheme with a weak-singlet seesaw, in a manner similar to the one described above, assumes $\tan\beta = v_t/v_b = 1$ (where $v_t, v_b = v_{EW}/\sqrt{2}$ are the VEVs in the top and bottom sector). This clearly implies that $\mu_{\chi\chi} = \mu_{\omega\omega}$ and $m_{t\chi} = m_{b\omega}$. This situation is illustrated in Fig. 3. Therefore $m_t/m_b = \mu_{t\chi}/\mu_{b\omega}$. The most stringent constraint comes from the parameter R_b and the limit on "bare" masses, $\mu_{\chi\chi} = \mu_{\omega\omega}$, is now pushed up to $12 - 15\text{TeV}$ [16] (this implies an even smaller window between Λ_{cr} and the heavy "bare" mass than in the previous case - by roughly a factor $3\sqrt{2}$)[7].

---

[6]Clearly, the dynamical mass may be then related to $\tan\beta = \frac{m_{t\chi}}{m_{b\omega}} = \frac{\Lambda_{cr} - \mu_{\chi\chi}}{\Lambda_{cr} - \mu_{\omega\omega}}$.

[7]As the heavy masses are very large here (setting the scale of mass generation dynamics) the effect of the Φ^4 term contribution to the effective mass may be safely neglected and the scalar loops, we believe, should definitely be considered.

4

2 Top-Bottom Color and Weak-Doublets Seesaw

We consider[8] an extended third generation quark sector with new weak-doublet quarks transforming under the SM gauge group $(SU(3)_{QCD} \otimes SU(2)_W \otimes U(1)_Y)$ as

$$\Psi_{1L} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, (3,2,1/6); \qquad \Psi_{2L} = \begin{pmatrix} T_L \\ B_L \end{pmatrix}, (3,2,1/6);$$

$$\Psi_R = \begin{pmatrix} T_R \\ B_R \end{pmatrix}, (3,2,1/6); \quad t_R, (3,1,2/3); \quad b_R, (3,1,-1/3). \qquad (2.1)$$

The mass matrices in the top and bottom sectors are

$$(\bar{t}_L \;\; \overline{T}_L) \begin{pmatrix} m_1 & m_p \\ 0 & m_q \end{pmatrix} \begin{pmatrix} t_R \\ T_R \end{pmatrix} \qquad (2.2)$$

$$(\bar{b}_L \;\; \overline{B}_L) \begin{pmatrix} m_2 & m_p \\ 0 & m_q \end{pmatrix} \begin{pmatrix} b_R \\ B_R \end{pmatrix}. \qquad (2.3)$$

At this point we imagine m_p and m_q to be the "bare", weak doublet-doublet, mass terms and m_1 and m_2 to be the (Topcolor) dynamical, weak singlet-doublet, mass terms. In addition we assume $m_p > m_q > m_1 > m_2$. Performing separately the rotations of the lefthanded (by angle φ_q^L) and righthanded (by angle φ_q) fermions the above mass matrices are diagonalized and we obtain the physical light and heavy masses, i.e.

$$m_t \approx \frac{m_1 m_q}{\sqrt{m_p^2 + m_q^2}} \;\; ; \;\; m_b \approx \frac{m_2 m_q}{\sqrt{m_p^2 + m_q^2}} \;\; ; \;\; M_T \approx M_B \approx M = \sqrt{m_p^2 + m_q^2} \qquad (2.4)$$

The EW precision measurements yield a 3σ limit on the righthanded bottom mixing - we find [18] $\sin^2 \varphi_b < 0.0052$ while known physical top and bottom quark masses yield a more stingent[9] consistency limit [18], i.e. $\sin^2 \varphi_b < 0.0006$.

---

[8]We are not first to introduce this extended third generation seesaw structure with weak-doublet quarks in the Topcolor type of models. As we learned at some point after we presented this material at the *Thinkshop*[2], an identical low-energy structure with weak-doublet quarks has been introduced in the spirit of a third generation specific TC^2 strong dynamical scheme (with identical extended Topcolor gauge group sector but with $v_t \approx v_{EW}$) in the work of the He, Tait and Yuan, [17]. We thank H.-J. He for informing us about his past work. The main differences between our strong dynamics scenario (Top-Bottom Color) and the one in reference [17] are: 1) instead of using a strong $U(1)$ gauge group - that necessarily has to be very strong to avoid a fine tuning problem (and therefore, we believe, it is not easily accommodated in a natural dynamical scheme) - we introduced the top-bottom mass splitting through the effect of an additional strong, asymptotically free, non-abelian $SU(3)$ gauge group, and 2) we suggest the bottom quark mass generation via the same Topcolor type of mechanism while reference [17] suggests that the bottom quark mass must be generated by different mechanism.

[9]The stringent limit [18] is obtained by assuming the Topcolor value $m_1 = 600$GeV, giving $\sin^2 \varphi_b < 10^{-5}$.
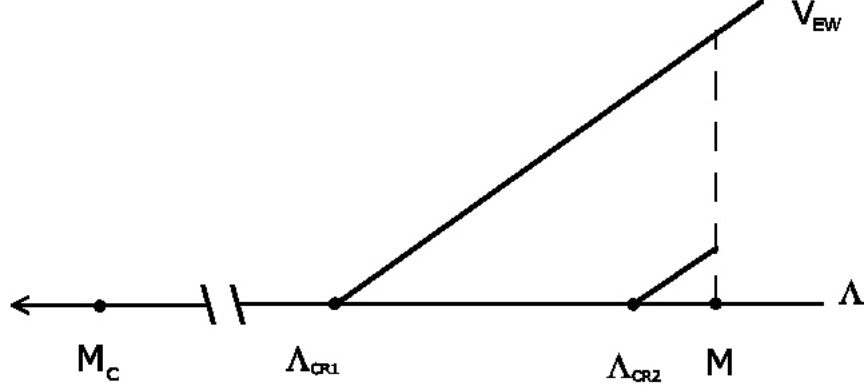
Figure 4: The schematic illustration of the mass renormalization curves in the Top-Bottom Color scenario.

Clearly, both the top and bottom sectors have the same infrared cut-off ($\sim M$) as illustrated in Fig. 4. Therefore, one needs different NJL interactions in the top and bottom sectors in order to introduce the isospin mass splitting. As shown in Fig. 4 two different "critical" energy scales, Λ_{cr1} and Λ_{cr2} are present, corresponding to the top and bottom mass renormalization curves respectively.

The third generation isospin splitting is introduced [18] via a strong $SU(3)$ gauge group in addition to the Topcolor structure (therefore the name - Top-Bottom Color). The third generation quark sector introduced in the low energy setup above transforms under this extended gauge structure as

$$\Psi_{1L} = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, (1,3,1,2,1/6); \quad \Psi_{2L} = \begin{pmatrix} T_L \\ B_L \end{pmatrix}, (1,1,3,2,1/6);$$

$$\Psi_R = \begin{pmatrix} T_R \\ B_R \end{pmatrix}, (1,1,3,2,1/6); \quad t_R, (1,3,1,1,2/3); \quad b_R, (3,1,1,1,-1/3); \quad (2.5)$$

where instead of gauge assignments under $SU(3)_{QCD}$, as in equation (2.1), we introduce the assignments under the strong $SU(3)$ gauge "trio". The schematic illustration of the cascade of symmetry breakings and fermion charge assignments under the strong $SU(3)$ gauge groups is shown in Fig. 5.

Defining the gauge couplings of this strong "trio" as g_1, g_2 and g_3 we find the tilting interaction below the higher symmetry breaking scale Λ_1 (corresponding to $SU(3)_1 \otimes SU(3)_2 \rightarrow SU(3)_{1'}$ and giving an octet of *precolorons*, i.e. heavier colorons) to be proportional to

$$\frac{g_2^2}{M_{pc}^2(g_1^2 + g_2^2)} \left[g_2^2 \left(\overline{\Psi}_{1L} t_R \right) \left(\overline{t}_R \Psi_{1L} \right) - g_1^2 \left(\overline{\Psi}_{1L} b_R \right) \left(\overline{b}_R \Psi_{1L} \right) \right] . \qquad (2.6)$$
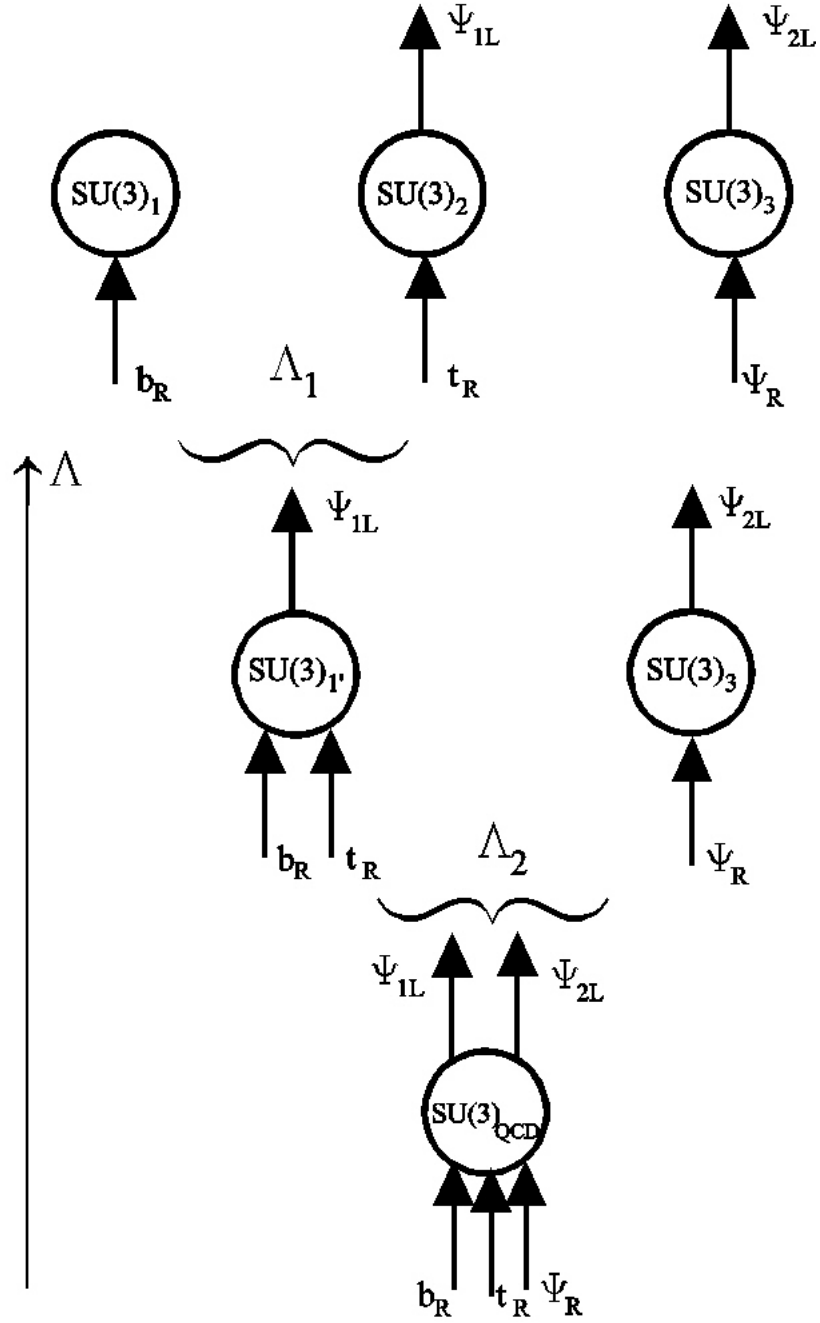
Figure 5: The schematic illustration of the cascade of symmetry breakings and fermion charge assignments under the strong $SU(3)$ gauge groups in the Top-Bottom Color scenario.

where M_{pc} is the precoloron mass. However, we assume that these interactions are not strong enough to trigger dynamical condensation. Nonetheless, they represent a crucial tilting needed for isospin splitting of the third generation quarks.

The coupling $g_{1'}$ (the $SU(3)_{1'}$ gauge coupling) runs enough below Λ_1 so that the NJL interactions triggered at the lower scale Λ_2 (corresponding to $SU(3)_{1'} \otimes SU(3)_3 \to SU(3)_{QCD}$ breaking) are strong enough to produce condensates in both the top and bottom sectors[10] in the standard manner of Topcolor models. Therefore, we obtain an effective composite two Higgs doublet model with $\tan \beta = m_t/m_b$.

Addendum The generation of bottom (in addition to top) quark mass in Topcolor models certainly introduce a new amount of fine tuning in the theory. Whether this represents a drawback for model structure or the natural consequence of an additional parameter in the theory is yet to be understood. Certainly, the fine tuning is much smaller than with a triggering scale of order the Planck mass (and the creation of the top quark mass alone). Anyhow, we find that as a lesser problem than the missing explanation of *why the top Yukawa coupling equals one*. Stated differently, the successful seesaw model, for example, definitely needs to explain in a natural manner why the "bare" masses relate *conspiratorially* when the dynamical mass has the fixed value of say $O(600\text{GeV})$. The possibility that the bottom mass (in our case dynamical mass $m_2 \geq 12\text{GeV}$) may have a different origin - either through ETC contributions, Yukawa couplings with composite *top* Higgs scalar (from the higher dimensional effective operators) or instanton contributions[11] - should be carefully considered as well.

References

[1] K. G. Wilson, unpublished; quoted in L. Susskind, Phys. Rev. **D20**, 2619 (1979); G. 't Hooft, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft at al. , *Plenum* New York, (1980).

[2] S. Weinberg, Phys. Rev. D **13**, 974 (1976); **19**, 1277 (1979); L. Susskind, *ibid.* **20**, 2619 (1979).

[3] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979); E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).

---

[10]A more detailed description of this dynamics may be found in reference [18], illustrated with an anomaly free, example model structure.

[11]We thank C. T. Hill for pointing us to the importance of this possibility.

[4] R. S. Chivukula, Lectures presented at 1997 Les Houches Summer School, [hep-ph/9803219].

[5] V. A. Miransky, M. Tabanashi and K. Yamawaki, Phys. Lett. B **221**, 177 (1989); Mod. Phys. Lett. A **4**, 1043 (1989); Y. Nambu, Chicago report EFI 89-08 (1989); W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989); Phys. Rev. D **41**, 219 (1990).

[6] W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. **D 41**, 1647 (1990).

[7] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[8] H. Pagels and S. Stokar, Phys. Rev. **D20**, 2947 (1979).

[9] C. T. Hill, Phys. Lett. B **266**, 419 (1991).

[10] R. S. Chivukula, A. G. Cohen and E. H. Simmons, Phys. Lett. B **380**, 92 (1996).

[11] C. T. Hill, Phys. Lett. B **345**, 483 (1995).

[12] K. Lane and E. Eichten, Phys. Lett. B **352**, 382 (1995); K. D. Lane, Phys. Rev. D **54**, 2204 (1996); G. Buchalla *et al.*, Phys. Rev. D **53**, 5185 (1996).

[13] M. B. Popovic and E. H. Simmons, Phys. Rev. **D58**, 095007 (1998) [hep-ph/9806287].

[14] B. A. Dobrescu and C. T. Hill, Phys. Rev. Lett. **81**, 2634 (1998) [hep-ph/9712319].

[15] R. S. Chivukula, B. A. Dobrescu, H. Georgi and C. T. Hill, Phys. Rev. **D59**, 075003 (1999) [hep-ph/9809470].

[16] H. Collins, A. Grant and H. Georgi, Phys. Rev. **D61**, 055002 (2000) [hep-ph/9908330].

[17] H.-J. He, T. M. P. Tait and C.-P. Yuan, Phys. Rev. **D62**, 011702 (2000) [hep-ph/9911266].

[18] M. B. Popovic, (2000) [hep-ph/0101123].